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Intermediate structure in nucleon–nucleus scattering†

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Abstract. The problem of inelastic nucleon–nucleus scattering is formulated in the framework of Green functions. In this microscopic description, the non-instantaneous (energy-dependent) part of the effective particle–hole force (irreducible vertex) determines the non-direct contributions to the scattering matrix. In order to obtain the intermediate structure this vertex is studied in more detail and several approximations are given. Couplings of virtual phonons of different kinds play an important role in this procedure. If one goes beyond perturbation theory three different effective forces occur in the coupling process. The connection to microscopic nuclear structure calculations is discussed. The general form of the T matrix emerging from the structure of the effective particle–hole force is given. Additionally more explicit expressions are derived by assuming certain structures of the final nucleus state.

1. Introduction

The aim of this paper is to describe inelastic nucleon–nucleus scattering within the framework of Green functions. This seems to be a useful tool to investigate scattering processes, since one can easily link the S matrix for general scattering processes with boundary values of Green functions (Naminiki 1960, Zhivopistsev 1965, Villars 1967). The theory of Green functions (Abrikosov *et al* 1965, Schultz 1964, Migdal 1965, Brown 1972, Fetter and Walecka 1971, Mattuck 1967) on the other hand is an established microscopic many-body theory, which can deal in a transparent manner with effective quantities (Migdal 1965, Brown 1972, Fetter and Walecka 1971, Mattuck 1967)—nowadays essential in the description of nuclear phenomena. The scattering process, which has in a certain sense the simplest structure, is the nucleon scattering on one-hole nuclei, since in this case one has to treat essentially an extension of the (renormalized) RPA approach (or TD approach) to scattering processes. This problem has been studied in several publications (Mahaux and Weidenmüller 1969, Dietrich and Hara 1968, Dietrich and Dover 1969, Ginnocchio *et al* 1970, Weigel 1969, Wegmann 1969, Vogeler and Weigel 1973). The next step seems to be the investigation of nucleon scattering on even–even nuclei, for which we expect the intermediate coupling of phonons to the one-particle propagation to play (besides direct scattering) a role in the interpretation of cross sections (Love and Satchler 1967, Love 1969). Scattering processes of this type have been mostly investigated in the 'standard' DWBA approximation and the antisymmetrized DWBA approach (Hodgson 1971, Austern 1970, Young 1968, Jackson 1970). It is well

† A summary of this paper has been published in the Proceedings of the International Conference on Nuclear Physics, Munich 1973.

known that the validity of the standard DWBA is limited and we shall not repeat all the arguments in this paper (Emrich 1971). We will return to this question later after having formulated the problem. Therefore higher-order processes have been taken into account† in a more or less phenomenological manner (Hodgson 1971, Austern 1970, Young 1968, Jackson 1970, Emrich 1971, Geramb 1972, Geramb *et al* 1972, Geramb 1973). A recent semi-phenomenological analysis has been quite successful in explaining the data (Geramb 1972, Geramb *et al* 1972, Geramb 1973). The investigation was based on the theory of Love and Satchler (1967) (see also Love 1969), which amounts to an extension of the treatment of Villars (1967) taking first order (to the Hartree–Fock structure) correlations in the initial and final target state into account. The treatment is done in the standard antisymmetrized hamiltonian approach to the scattering process, replacing the potential by the commutator $[v, a_k]_-$. In our method the formulation via the effective particle–hole force (irreducible vertex) (Abrikosov *et al* 1965, Migdal 1965, Baym and Kadanoff 1961, Brenig and Wagner 1963) is used, which contains the essential intermediate stages of the process. It turns out that this quantity determines the features of the scattering (Emrich 1971) and suitable approximations are needed. The advantage of this formulation is that one can use methods and results already known from the nuclear structure problem. The role played by effective quantities of several types can be recognized and one has additionally a theory available to determine these quantities either from first principles or by linking them to known problems.

In § 2 we shall give necessary definitions and the general formulation of the problem in the Green function scheme. In § 3 we deal with approximations for the effective particle–hole force needed for the solution of the relevant equations. Section 4 is devoted to the structure of the scattering process emerging from the approximations for the irreducible vertex, the general form of the asymmetric one-particle propagator and the assumed structure of the excited final state of the nucleus.

2. General theory

The connection between the S matrix (T matrix) and the Green functions rests on a well known limiting process using the exact definitions of scattering states and Green functions (Naminiki 1960, Zhivopistsev 1965, Villars 1967). For completeness we will repeat the main steps of the procedure. For nucleon–nucleus scattering the required initial (final) scattering states are given as (Naminiki 1960, Gell-Mann and Goldberger 1953):

$$|\Omega_i^{(+)}\rangle = \frac{i\eta_i}{\epsilon_i + E_0 - H + i\eta_i} a_i^\dagger |0\rangle \, e^{i\delta_i} \equiv e^{i\delta_i} |\bar{\Omega}_i^{(+)}\rangle, \tag{2.1}$$

$$|\Omega_f^{(-)}\rangle = \frac{i\eta_f}{\epsilon_f + E_N - H - i\eta_i} a_f^\dagger |N\rangle \, e^{-i\delta_f} \equiv e^{-i\delta_f} |\bar{\Omega}_f^{(-)}\rangle. \tag{2.2}$$

Here $|0\rangle(|N\rangle)$ denotes the exact ground state (excited state) of the target:

$$H|0\rangle = E_0|0\rangle, \tag{2.3}$$

$$H|N\rangle = E_N|N\rangle. \tag{2.4}$$

† A recent extensive list of references is given in the habilitation of H V Geramb.

We restrict ourselves to bound states of the target nucleus. The total hamiltonian of the system is given by:

$$H = \sum_{\alpha\beta} \left\langle \alpha \left| \frac{p^2}{2m} \right| \beta \right\rangle a_\alpha^\dagger a_\beta + \tfrac{1}{2} \sum_{\alpha\beta} \sum_{\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma . \tag{2.5}$$

The Schrödinger creation (annihilation) operators $a_\alpha^\dagger(a_\alpha)$ of nucleons are defined with the standing wave boundary condition. By adding the phase factors $e^{i\delta_i}(e^{-i\delta_f})$ we ensure the correct asymptotic behaviour. It is suitable to choose for the single-particle states $\alpha\,(= \epsilon_\alpha, j_\alpha, l_\alpha, m_\alpha)$ a basis defined by an appropriate shell-model hamiltonian h:

$$h|\epsilon_\alpha\rangle = \epsilon_\alpha |\epsilon_\alpha\rangle. \tag{2.6}$$

With the help of model states defined according to the procedure of Villars (1967) by:

$$|\phi_{\alpha,N}^{(+)}\rangle = a_\alpha^\dagger |N\rangle\, e^{i\delta_\alpha} \equiv e^{i\delta_\alpha} |\bar{\phi}_{\alpha,N}^{(+)}\rangle, \tag{2.7}$$

we can express the required transition element as follows:

$$\begin{aligned} \langle \phi_{f,N}^{(-)} | \Omega_i^{(+)} \rangle &= e^{i(\delta_i + \delta_f)} \left(\delta_{N,0}\delta_{if} + \frac{1}{E_i - E_0 + i\eta} \langle (H - E_f) a_f^\dagger | N \rangle | \bar{\Omega}_i^{(+)} \rangle \right) \\ &= e^{i(\delta_i + \delta_f)} \left(\delta_{N,0}\delta_{if} + \frac{1}{E_i - E_f + i\eta} \langle \bar{\phi}_{f,N}^{(-)} | T | \bar{\phi}_{i,0}^{(+)} \rangle \right). \end{aligned} \tag{2.8}$$

The left-hand side of equation (2.8) can be expressed by the asymmetric Green function, defined as:

$$g_{\rho\delta}^{N0}(\omega) = \left\langle N \left| \left[a_\rho \left(\omega - H + \frac{E_N + E_0}{2} + i\eta \right)^{-1} a_\delta^\dagger + a_\delta^\dagger \left(\omega + H - \frac{E_N + E_0}{2} - i\eta \right)^{-1} a_\rho \right] \right| 0 \right\rangle, \tag{2.9}$$

where

$$g_{\rho\delta}^{N0}(\epsilon_r, \epsilon_d) = g_{\rho\delta}^{N0}\left(\frac{\epsilon_r + \epsilon_d}{2} \right) \delta(\epsilon_r - \epsilon_d + E_N - E_0) \tag{2.10}$$

is the Fourier transform of

$$g_{rd}^{N0} = -i\langle N | T(a_\rho(t_\rho) a_\delta^\dagger(t_d)) | 0 \rangle. \tag{2.11}$$

By comparison of (2.1), (2.7) and (2.9) we obtain the relation:

$$\langle \phi_{f,N}^{(-)} | \Omega_i^{(+)} \rangle = e^{i(\delta_i + \delta_f)} i\eta_i g_{fi}^{N0} \quad \left(\omega = \epsilon_i + \frac{E_0 - E_N}{2} \right) \tag{2.12}$$

In order to get the T matrix one has in the next step to put the equation of motion for g_{fi}^{N0} in such a form that the structure of the right-hand side of equation (2.8) emerges. Since the theory of normal Green functions, defined by

$$g_{a_1 \ldots a_n, b_1 \ldots b_n} = (-i)^n \langle 0 | T(a_{\alpha_1}(t_{a_1}) \ldots a_{\alpha_n}(t_{a_n}) a_{\beta_n}^\dagger(t_{b_n}) \ldots a_{\beta_1}^\dagger(t_{b_1})) | 0 \rangle, \tag{2.13}$$

has been studied in more detail (Abrikosov *et al* 1965, Schultz 1964, Migdal 1965, Brown 1972, Fetter and Walecka 1971, Mattuck 1967), it seems appropriate to connect directly the problem of the asymmetric Green function to the four-point function. This can be done utilizing the projection procedure, since the asymmetric Green function is contained in the four-point function. One obtains the following equation of motion

(W Brenig, private communication, Goldberger and Watson 1964, Speth 1970):

$$g^{N0}_{mn} = \delta_{N0} g_{mn} - \mathrm{i} g_{ma} g_{bn} I_{adbr} g^{N0}_{rd} \tag{2.14}$$

We use the convention, according to which summation or integration, respectively, is to be carried out over all doubly occurring (Latin) indices. I_{adbr} is the effective particle–hole interaction (irreducible vertex), defined via the effective single-particle potential u (irreducible mass operator) by (Baym and Kadanoff 1961, Brenig and Wagner 1963):

$$I_{adbr} = \mathrm{i} \frac{\delta u_{ab}}{\delta g_{rd}}. \tag{2.15}$$

The parts of the theory of the normal Green functions needed in our approach are given in the appendix. In energy space equation (2.14) becomes:

$$g^{N0}_{\mathrm{fi}}\left(\epsilon_{\mathrm{i}} + \frac{E_0 - E_N}{2}\right)$$

$$= \delta_{N0} g_{\mathrm{fi}}(\epsilon_{\mathrm{i}}) - \mathrm{i}(1-\delta_{N,0}) \sum_{\alpha\beta} \left[\mathrm{i} g_{f\alpha}(\epsilon_{\mathrm{i}} + E_0 - E_N) g_{\beta\mathrm{i}}(\epsilon_{\mathrm{i}}) \right.$$

$$\left. \times \sum_{\delta\rho} \int \mathrm{d}\omega \, I_{\alpha\delta\beta\rho}\left(\epsilon_{\mathrm{i}} + \frac{E_0 - E_N}{0}, \omega_{\mathrm{i}}; E_0 - E_N\right) g^{N0}_{\rho\delta}(\omega) \right]. \tag{2.16}$$

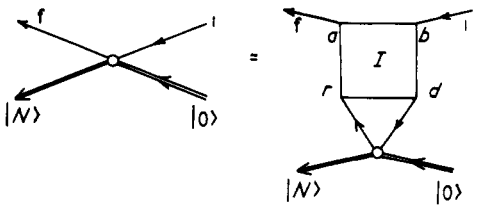
For equation (2.8) we obtain now:

$$\mathrm{e}^{-\mathrm{i}(\delta_{\mathrm{i}} + \delta_{\mathrm{f}})} \langle \phi^{(-)}_{\mathrm{f},N} | \Omega^{(+)}_{\mathrm{i}} \rangle = \mathrm{i}\eta_{\mathrm{i}} g^{N0}_{\mathrm{fi}}\left(\epsilon_{\mathrm{i}} + \frac{E_0 - E_N}{2}\right)$$

$$= \delta_{N0} \mathrm{i}\eta_{\mathrm{i}} g_{\mathrm{fi}}(\epsilon_{\mathrm{i}}) - (1-\delta_{N0}) \sum_{\alpha\beta} \left[g_{f\alpha}(\epsilon_{\mathrm{i}} + E_0 - E_N) \mathrm{i}\eta_{\mathrm{i}} g_{\beta\mathrm{i}}(\epsilon_{\mathrm{i}}) \right.$$

$$\left. \times \sum_{\delta\rho} \int \mathrm{d}\omega \, I_{\alpha\delta\beta\rho}\left(\epsilon_{\mathrm{i}} + \frac{E_0 - E_N}{2}, \omega; E_0 - E_N\right) g^{N0}_{\rho\delta}(\omega) \right]. \tag{2.17}$$

Equations (2.14) or (2.16), which determine the scattering process, exhibit the formal structure of the scattering. If we express these equations graphically, we obtain:



(2.14)

The incoming particle, described by the one-particle propagator $g_{b\mathrm{i}}$, propagates in the medium until it scatters in matter via the irreducible vertex I causing a stable excitation of the nucleus (g^{N0}_{rd}). After this process the one-particle propagation takes place to the final state f. The whole process can therefore be split up into three steps: First, the incoming particle, asymptotically in the shell-model state i, distributes its single-particle strength among several shell-model states (non-diagonal single-particle propagator) due to the interaction with virtual phonons corresponding to 'elastic' scattering. The

same can happen for the outgoing particle. These shell-model states, properly weighted, interact in the intermediate step due to virtual processes contained in I in the presence of the final stable excitation of the nucleus. The investigation of I is the topic of the next section.

The treatment of the single-particle propagation goes as follows.

Using the basis of the hamiltonian h the two-point function can be cast in the form:

$$g_{\alpha\beta}(\epsilon) = g^{s}_{\alpha\beta}(\epsilon) + \sum_{\mu\nu} g^{s}_{\alpha\mu}(\epsilon)\tau^{D}_{\mu\nu}(\epsilon)g^{s}_{\nu\beta}(\epsilon), \tag{2.18}$$

where $g^{s}_{\alpha\beta}(\epsilon)$ has the shell-model structure:

$$g^{s}_{\alpha\beta}(\epsilon) = \delta_{\alpha\beta}g^{s}_{\alpha}(\epsilon) = \delta_{\alpha\beta}\left(\frac{1-n_\alpha}{\epsilon-\epsilon_\alpha+i\eta}+\frac{n_\alpha}{\epsilon-\epsilon_\alpha-i\eta}\right). \tag{2.19}$$

The reducible mass operator τ^{D} obeys the equation

$$\begin{aligned}\tau^{D}_{\alpha\beta}(\epsilon) &= u^{D}_{\alpha\beta}(\epsilon) + \sum_{\mu} u^{D}_{\alpha\mu}(\epsilon)g^{s}_{\mu}(\epsilon)\tau^{D}_{\mu\beta}(\epsilon) \\ &= u^{D}_{\alpha\beta}(\epsilon) + \sum_{\mu\nu} u^{D}_{\alpha\mu}(\epsilon)g_{\mu\nu}(\epsilon)u^{D}_{\nu\beta}(\epsilon).\end{aligned} \tag{2.20}$$

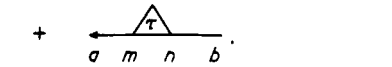
u^{D} is the energy-dependent difference between the full u and the shell-model potential u^{s}. It causes no difficulties to include an imaginary part in the shell-model potential. The expression $i\eta_i g_{\beta i}(\epsilon_i)$ is by definition the single-particle wavefunction for elastic scattering. In an obvious notation we obtain:

$$\begin{aligned}e^{i(\delta_i+\delta_\beta)}i\eta_i g_{\beta i}(\epsilon_i) &= \left\langle 0\left| a_\beta \frac{i\eta_i}{\epsilon_i-(H-E_0)+i\eta_i} a^{\dagger}_i \right| 0 \right\rangle e^{i(\delta_i+\delta_\beta)} \\ &= \langle 0;\beta^{(-)}|N+1,i^{(+)}\rangle = \varphi^{(+)}_\beta(\epsilon_i) = \bar{\varphi}^{(+)}_\beta(\epsilon_i)\,e^{i(\delta_i+\delta_\beta)}.\end{aligned} \tag{2.21}$$

The equation for $\bar{\varphi}$ can be read off from (2.18):

$$\bar{\varphi}^{(+)}_\beta(\epsilon_i) = \delta_{\beta i} + g^{s}_{\beta}(\epsilon_i)\tau^{D}_{\beta i}(\epsilon_i). \tag{2.22}$$

τ^{D} describes the deviations in elastic scattering mainly due to couplings to virtual phonons of several types. Formally (2.18) is expressed graphically as follows:

$g_{ab} =$  (2.23)

Now insertion of (2.18) and (2.21) into (2.17) yields:

$$\begin{aligned}\langle\bar{\phi}^{(-)}_{f,N}|T|\bar{\phi}^{(+)}_{i,0}\rangle = \Bigg\{ &\delta_{N,0}\tau^{D}_{fi}(\epsilon_i) - (1-\delta_{N,0})i\sum_{\alpha\beta}\sum_{\delta\rho}\Bigg[(\delta_{f\alpha}+\tau^{D}_{f\alpha}(\epsilon_f)g^{s}_{\alpha}(\epsilon_f))(\delta_{\beta i}+g^{s}_{\beta i}(\epsilon_i)\tau^{D}_{\beta i}(\epsilon_i)) \\ &\times \int d\omega\, I_{\alpha\delta\beta\rho}\left(\epsilon_i+\frac{E_0-E_N}{2},\omega;E_0-E_N\right)g^{N0}_{\rho\delta}(\omega)\Bigg]\Bigg\} = \bar{T}_{fN,i0}.\end{aligned} \tag{2.24}$$

So, the incoming (outgoing) particle can distribute its single-particle strength among several shell-model states due to the interaction with the core corresponding to elastic scattering. The inclusion of the single-particle potential beyond the Hartree–Fock approximation in calculating the two-point function is intimately related to the inelastic

scattering, because I directly emerges from u according to (2.15). This question has been attacked since the pioneer work Brown *et al* (1963) in different approximations (MacKellar *et al* 1971, Dover and Giai 1971, Bruneau and Vinh-Mau 1970, Werner 1967, Brenig 1967, Weigel 1968, Zawischa and Werner 1969, Vinh-Mau 1970). The connection with higher-order response functions has also been studied (Winter 1972). Since we will encounter similar problems in constructing I we will not discuss this problem further and assume, if necessary, that in some approximation the two-point function (or τ^{D}) is known. In general it is possible that resonances of the elastic scattering, due to the occurrence of τ^{D}, also show up in the inelastic process (see § 4).

If I is assumed to act instantaneously one obtains necessarily the DWBA approximation:

$$g^{N0}_{\mathrm{fi,DWBA}}(\omega) = -2\pi\mathrm{i}\sum_{\alpha\beta}\sum_{\delta\rho} L^{0}_{\mathrm{f}\beta\mathrm{i}\alpha}(\omega, E_0 - E_N) I^{(\mathrm{i})}_{\alpha\delta\beta\rho}\langle N|a^{\dagger}_{\delta}a_{\rho}|0\rangle, \tag{2.25}$$

where L^0 denotes the uncorrelated response function:

$$L^{0}_{\alpha\beta\gamma\rho}(\omega, \Omega) \equiv \mathrm{i}g_{\alpha\rho}\left(\omega+\frac{\Omega}{2}\right) g_{\beta\gamma}\left(\omega-\frac{\Omega}{2}\right). \tag{2.26}$$

Restriction to shell-model single-particle propagation leads to the 'standard' DWBA approximation:

$$T^{\mathrm{DWBA}}_{\mathrm{f}N,\mathrm{i}0} = 2\pi\sum_{\rho\delta} I_{\mathrm{f}(-)\delta\mathrm{i}(+)\rho}\langle N|a^{\dagger}_{\delta}a_{\rho}|0\rangle, \tag{2.27}$$

which agrees with the result of Love (1969, equation (41)), if we assume a Hartree–Fock structure for $|N\rangle$. The second term of (41) would correspond to a non-diagonal one-particle propagator. It is obvious from the comparison of the spectral representations that the DWBA approximation (2.25) adds additional poles to g^{N0}_{fi}, which are incorrect. The pole structure of (2.25) coincides only partly with the correct structure (2.9). Whether the DWBA approximation holds depends on the problem under consideration (Hodgson 1971, Austern 1970, Young 1968, Jackson 1970, Emrich 1971, Geramb 1972, Geramb *et al* 1972, Geramb 1973). If one wants to go beyond the DWBA non-instantaneous terms of I must be included, which is discussed in the next section.

Formally one can obtain a resonance structure in the scattering amplitude by splitting up the effective particle–hole force in the instantaneous and non-instantaneous part. One gets by iteration of (2.16) the following result ($\Omega \equiv E_0 - E_N$):

$$g^{N0}_{\mathrm{fi}}(\omega) = g^{N0}_{\mathrm{fi,DWBA}}(\omega) - \sum_{\alpha\beta}\sum_{\rho\delta}\int L^{0}_{\mathrm{f}\beta\mathrm{i}\alpha}(\omega, \Omega) T^{(\mathrm{ni})}_{\alpha\delta\beta\rho}(\omega, \omega'; \Omega) g^{N0}_{\rho\delta,\mathrm{DWBA}}(\omega')\,\mathrm{d}\omega'. \tag{2.28}$$

$T^{(\mathrm{ni})}$ obeys the equation:

$$T^{(\mathrm{ni})}_{\alpha\delta\beta\rho}(\epsilon, \epsilon'; \omega) = I^{(\mathrm{ni})}_{\alpha\delta\beta\rho}(\epsilon, \epsilon'; \omega) - \int \mathrm{d}\epsilon'' \left(\sum_{\xi\eta}\sum_{\gamma\delta} I^{(\mathrm{ni})}_{\alpha\zeta\beta\eta}(\epsilon, \epsilon''; \omega) L^{0}_{\eta\tau\xi\gamma}(\epsilon'', \omega) T^{(\mathrm{ni})}_{\gamma\delta\tau\rho}(\epsilon'', \epsilon'; \omega)\right). \tag{2.29}$$

Insertion of (2.28) into (2.17) leads to the standard decomposition of the scattering process into the direct plus compound term.

As we shall see in the next section, only approximate expressions for I can be obtained. Even if we were to assume a more or less rigorous knowledge of I, one has still to solve

equation (2.14) or (2.16), respectively. Since equation (2.16) is an integral equation with respect to the energy, for which solutions are not known, one has to discuss several simplifications in order to obtain a tractable problem. This will be postponed to § 4.

3. Approximations for the irreducible vertex

It has been shown in § 2 that the first relevant question consists in finding an expression for the non-instantaneous (or equivalently the energy dependent) part of the effective particle–hole force. As known from similar problems in field theory only formal closed expressions exist for the irreducible vertex (see appendix). The reason is that due to the two-body force one cannot eliminate all correlations of higher order. One has either to know the functional derivative of the effective scattering amplitude Γ in the medium with respect to the one-particle propagator or the irreducible particle–hole vertex for the so-called six-point response function, respectively. Therefore one is restricted to suitable approximations for the effective particle–hole force in order to proceed with the solution for the asymmetric Green function. We are going to discuss essentially three approximations.

3.1. Perturbation expansion of I

The simplest choice is the discussion of the perturbation expansion, which can be obtained by iterating (A.1) and (A.2) starting with $\delta u/\delta q = 0$. Up to second order one obtains:

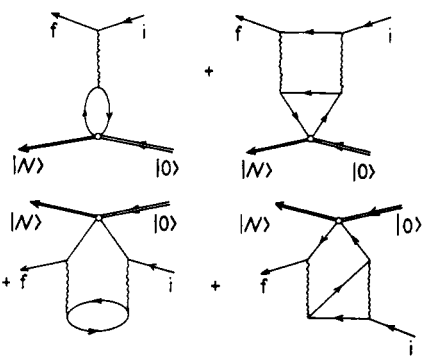
$$
\begin{aligned}
2\pi I_{\alpha\delta\beta\rho}(\epsilon, \epsilon'; \omega) \\
&\simeq 2v_{\alpha\delta\beta\rho} + 2\sum_{\zeta\eta}\sum_{\mu\nu}[v_{\alpha\delta\zeta\eta}\Lambda_{\zeta\eta\mu\nu}(\epsilon+\epsilon')v_{\mu\nu\beta\rho} + v_{\alpha\zeta\rho\eta}L^0_{\eta\mu\zeta\nu}(\epsilon-\epsilon')v_{\delta\nu\beta\mu} \\
&\quad + 2v_{\alpha\zeta\eta\rho}L^0_{\eta\mu\zeta\nu}(\epsilon-\epsilon')v_{\nu\delta\beta\mu}], \qquad (3.1)
\end{aligned}
$$

with

$$
L^0_{\eta\mu\zeta\nu}(\omega) = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}\epsilon\, g_{\eta\nu}\left(\epsilon+\frac{\omega}{2}\right) g_{\mu\zeta}\left(\epsilon-\frac{\omega}{2}\right), \qquad (3.2)
$$

$$
\Lambda_{\zeta\eta\mu\nu}(\omega) = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}\epsilon\, g_{\zeta\mu}\left(\epsilon+\frac{\omega}{2}\right) g_{\eta\nu}\left(\frac{\omega}{2}-\epsilon\right). \qquad (3.3)
$$

In this approximation one has schematically the following contributions for g^{NO}_{fi}:



(3.4)

The first graph corresponds to the DWBA approximation, in which the energy integration is trivial (see (2.25)).

3.2. Ladder approximation

In the second approximation, we assume that the effective single-particle potential is represented by the ladder approximation, which reduces with some additional neglections to the Brueckner approximation:

$$u_{kl} \simeq -\mathrm{i}\Gamma^{\mathrm{L}}_{kmln} g_{nm} \tag{3.5}$$

with

$$\begin{aligned}\Gamma^{\mathrm{L}}_{klmn} &= 2v_{klmn} + \mathrm{i}v_{klab} g_{ar} g_{bs} \Gamma^{\mathrm{L}}_{rsmn} \\ &= 2v_{klmn} + \mathrm{i}v_{klab} g^{\mathrm{L}}_{abrs} v_{rsmn}.\end{aligned} \tag{3.6}$$

One obtains then in this approximation by use of (2.15) and (3.6):

$$I^{\mathrm{L}}_{\alpha\delta\beta\rho}(\epsilon\epsilon'\omega) = \Gamma^{\mathrm{L}}_{\alpha\delta\beta\rho}(\epsilon+\epsilon') + \sum_{\zeta\eta}\sum_{\tau\mu}\int \mathrm{d}\Omega\, \Gamma^{\mathrm{L}}_{\alpha\zeta\rho\eta}\left(\Omega+\frac{\omega}{2}\right) \mathrm{i} g_{\eta\tau}(\Omega-\epsilon') g_{\mu\zeta}(\Omega-\epsilon) \Gamma^{\mathrm{L}}_{\delta\tau\beta\mu}\left(\Omega-\frac{\omega}{2}\right). \tag{3.7}$$

In energy space the formal solution of (3.6) reads as follows:

$$2\pi\Gamma^{\mathrm{L}}_{\alpha\delta\beta\rho}(\epsilon+\epsilon') = 2v_{\alpha\delta\beta\rho} + \sum_{\zeta\eta}\sum_{\bar{\zeta}\bar{\eta}} v_{\alpha\delta\zeta\mu} g_{\zeta\eta\bar{\zeta}\bar{\eta}}(\epsilon+\epsilon') v_{\bar{\zeta}\bar{\eta}\beta\rho}, \tag{3.8}$$

$g_{\alpha\beta\rho\mu}(\omega)$ is defined in (3.14). In Feynman graphs (3.7) exhibits a similar structure to the first two graphs of (3.4). One has only to replace the bare interaction by Γ^{L}. If we restrict ourselves to second order in the interaction we return to the result (3.1).

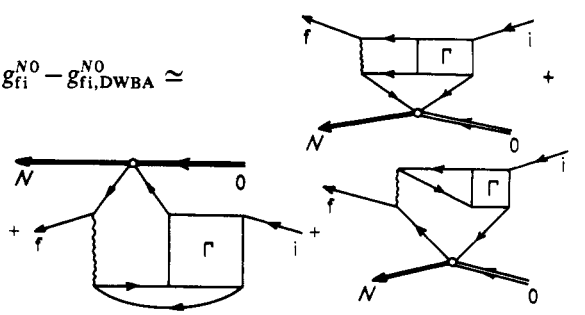
3.3. Linear approximation

The third approximation is the linear approximation for I neglecting the term $\delta\Gamma/\delta g$ in I (see A.5). One obtains (Brenig and Wagner 1963, Weigel 1972):

$$I_{adbr} \simeq 2v_{adbr} + \mathrm{i}v_{admn} g_{mk} g_{ns} \Gamma_{ksbr} + \mathrm{i}v_{manr} g_{ns} g_{km} \Gamma_{dsbk} - \mathrm{i}v_{manr} g_{ns} g_{km} \Gamma_{sdbk} \tag{3.9}$$

from which the following graphical representation for g^{NO}_{fi} results:

$g^{NO}_{\mathrm{fi}} - g^{NO}_{\mathrm{fi,DWBA}} \simeq$



(3.10)

Instead of treating equation (3.9) directly, which requires a knowledge of the structure of the reducible vertex, we are going to use the connection between the reducible vertex

(effective scattering amplitude in medium) and the four-point function (Winter 1972, Weigel 1972, Nozieres 1964):

$$g_{ax}g_{by}\Gamma_{xycd} = g_{abxy}K_{xycd}, \tag{3.11}$$

or

$$L_{axby}I_{ycxd} = \mathrm{i}g_{ay}g_{xb}\Gamma_{ycxd}. \tag{3.12}$$

The effective particle–particle force K is defined in the appendix (A.6). The structures of $g^{(2)}$ and L are more transparent as the structure of Γ (see 3.16, 3.17). L denotes the generalized response function defined in (A.1). It seems appropriate in the spirit of the linear approximation (3.9) to replace K and I by the nucleon–nucleon interaction and to use for $g^{(2)}$ (L) the outcome of the standard p–p RPA (p–h RPA) problem (Brown 1972, Fetter and Walecka 1971, Mattuck 1967, Rowe 1970). After performing this procedure we obtain†:

$$\begin{aligned}\sum_{\beta} I_{\alpha\delta\beta\rho}(\epsilon\epsilon', \Omega)g_{\beta\mathrm{i}}\left(\epsilon-\frac{\Omega}{2}\right) &\simeq \frac{1}{2\pi}\sum_{\beta}\Bigg\{\Bigg(2v_{\alpha\delta\beta\rho} + \sum_{\zeta\eta}\sum_{\mu\nu}(v_{\alpha\delta\zeta\eta}g_{\zeta\eta\mu\nu}(\epsilon+\epsilon')v_{\mu\nu\beta\rho} \\ &\quad + 2v_{\zeta\alpha\eta\rho}L_{\eta\mu\zeta\nu}(\epsilon-\epsilon')v_{\delta\nu\beta\mu}\Bigg)g_{\beta\mathrm{i}}\left(\epsilon-\frac{\Omega}{2}\right) \\ &\quad - 2v_{\zeta\alpha\eta\rho}\int \mathrm{d}\omega\, L_{\eta\beta\mathrm{i}\nu}\left(\epsilon-\frac{\Omega}{2}+\frac{\omega}{2}, \omega\right)g_{\mu\zeta}\left(\omega+\epsilon'-\frac{\Omega}{2}\right)v_{\nu\delta\beta\mu}\Bigg\}.\end{aligned} \tag{3.13}$$

Here, we have introduced the following definitions:

$$g_{\zeta\eta\mu\nu}(\omega) = \frac{\mathrm{i}}{2\pi}\oint\!\!\!\!\!\!\int \mathrm{d}\epsilon\, \mathrm{d}\epsilon'\, g_{\zeta\eta\mu\nu}(\epsilon\epsilon', \omega), \tag{3.14}$$

$$L_{\zeta\eta\mu\nu}(\omega) = \oint\!\!\!\!\!\!\int \mathrm{d}\epsilon\, L_{\zeta\eta\mu\nu}(\epsilon, \omega) = \oint\!\!\!\!\!\!\int \mathrm{d}\epsilon \oint\!\!\!\!\!\!\int \frac{\mathrm{d}\epsilon'}{2\pi}L_{\zeta\eta\mu\nu}(\epsilon\epsilon', \omega). \tag{3.15}$$

In equation (3.14) we split up the energy variables according the p–p channel, in (3.15) the p–h channel decomposition was used. For the spectral representation the following structure holds:

$$g_{\zeta\eta\mu\nu}(\omega) = -\left(\sum_{|N+2\rangle}\frac{\rho^{N+2}_{\zeta\eta}(\rho^{N+2}_{\nu\mu})^*}{\omega-(E^{N+2}-E^N_0)+\mathrm{i}\eta} - \sum_{|N-2\rangle}\frac{\rho^{N-2}_{\zeta\eta}(\rho^{N-2}_{\nu\mu})^*}{\omega-(E^N_0-E^{N-2})-\mathrm{i}\eta}\right), \tag{3.16}$$

$$L_{\zeta\eta\mu\nu}(\omega) = -\sum_{|N\rangle\neq|0\rangle}\left(\frac{\rho^N_{\zeta\mu}(\rho^N_{\nu\eta})^*}{\omega-(E^N-E^N_0)+\mathrm{i}\eta} - \frac{\rho^N_{\eta\nu}(\rho^N_{\mu\zeta})^*}{\omega+(E^N-E^N_0)-\mathrm{i}\eta}\right), \tag{3.17}$$

with

$$\rho^{N+2}_{\zeta\eta} = \langle 0|a_\zeta a_\eta|N+2\rangle; \qquad \rho^{N-2}_{\zeta\eta} = \langle N-2|a_\zeta a_\eta|0\rangle, \tag{3.18}$$

$$\rho^N_{\zeta\eta} = \langle 0|a^\dagger_\eta a_\zeta|N\rangle. \tag{3.19}$$

The amplitudes ρ can be taken from the p–p or p–h RPA problem, respectively. One should use in the RPA problem in this case the nucleon–nucleon interaction, eventually

† This result is also obtainable by application of the Martin–Schwinger procedure plus projection method to the equation of motion for the asymmetric Green function.

renormalized due to the restriction of the basis (see, for instance, Migdal 1965, Beres and MacDonald 1967, Austin and Crawley 1972). If one uses in the integral equation for L the instantaneous nucleon–nucleon interaction the integral in (3.13) reduces to (Weigel 1969):

$$\int d\omega\, L(\ldots)\, g(\ldots) = \int d\omega \left\{ g_{\eta\nu}\left(\epsilon - \frac{\Omega}{2}\right) g_{\mu\zeta}\left(\omega + \epsilon' - \frac{\Omega}{2}\right) g_{\beta i}\left(\epsilon - \frac{\Omega}{2}\right) - \sum_{\tau\delta}\sum_{\rho\lambda}\left[2g_{\eta\tau}\left(\epsilon - \frac{\Omega}{2} + \omega\right) g_{\rho i}\left(\epsilon - \frac{\Omega}{2}\right) g_{\mu\zeta}\left(\omega + \frac{\epsilon'}{2} - \frac{\Omega}{2}\right) v_{\tau\lambda\rho\delta}\, L_{\delta\beta\lambda\nu}(\omega)\right]\right\}. \quad (3.20)$$

3.4. Neglection of effective three-body interactions

Another possibility is the use of the correct expression for the particle–hole force in terms of the irreducible particle–hole force $K^{(ph)}$ belonging to integral equation for the correlated part of the six-point function g^c (Winter 1972):

$$g^c_{abc,\,mnr} = g_{abc,\,mnr} - g_{bcrj}g^{-1}_{js}g_{samn} + g^c_{bcrj}g^{-1}_{js}g^c_{samn}. \quad (3.21)$$

The correlated four-point function is defined by (A.6). For g^c the following equation holds:

$$g^c_{abc,\,mnr} = -i(L_{abmr}g_{cn} - L_{abnr}g_{cm}) - iL_{sbjr}g_{ct}[2v_{jtsd}(L - L^0)_{damn} + K^{(ph)}_{pjt,\,qws}g^c_{aqw,\,mnp}]. \quad (3.22)$$

It is possible, due to the restriction to two-body forces, to express L by g^c and v with the help of the equation of motion method. Comparison with the equation for L which contains the effective particle–hole force I as the integral kernel (see A.1) yields the following exact equation for I:

$$I_{adbr} = 2v_{adbr} + \tfrac{1}{2}i(K^{(ph)}_{mda,\,jtr} - K^{(ph)}_{mda,\,tjr})g_{jx}g_{tc}\Gamma_{cxby}g_{ym}. \quad (3.23)$$

The first term describes in the strict sense the antisymmetrized direct scattering, the second term takes care of polarization effects. If one neglects effective 'three-particle' forces (connected part of $K^{(ph)}$), corresponding for instance to anharmonic virtual states, the following approximation emerges:

$$K^{(ph)}_{mda,\,jtr} \simeq g^{-1}_{mr}K_{dajt} - g^{-1}_{dj}I_{matr}. \quad (3.24)$$

Introduction of the last approximation into equation (3.23) yields:

$$I_{adbr} \simeq 2v_{adbr} + iK_{admn}g_{mx}g_{ny}\Gamma_{xybr} - \tfrac{1}{2}iI_{matr}g_{tc}g_{ym}\Gamma_{cdby} + \tfrac{1}{2}iI_{majr}g_{jx}g_{ym}\Gamma_{dxby}$$
$$= 2v_{adbr} + iK_{admn}g_{mnxy}K_{xybr} + \tfrac{1}{2}I_{majr}L_{yjxm}I_{dxby} - \tfrac{1}{2}I_{matr}L^{(0)}_{tymc}I_{cdby} + O(I^3). \quad (3.25)$$

Equation (3.25) reduces to (3.9), if we insert on the right-hand side of (3.25) for I and K the first-order approximation. In the spirit of our philosophy we want to determine the pole structure of I, which can be approximately obtained by using the energy-independent approximation for I and K on the right-hand side of (3.25). The procedure agrees now exactly with the method used in evaluating equation (3.9). One has only to replace v by the energy-independent approximations for I and K. The p–h and p–p RPA should now be done with these forces (Migdal 1965, Bes and Broglia 1971). We obtain instead

of (3.13) the following structure:

$$\sum_{\beta} I_{\alpha\delta\beta\rho}(\epsilon,\epsilon';\Omega)g_{\beta i}\left(\epsilon-\frac{\Omega}{2}\right) = \frac{1}{2\pi}\sum_{\beta}\left\{\left(2v_{\alpha\delta\beta\rho}+(2\pi)^2\sum_{\mu\nu}\sum_{\zeta\eta}[K^{(s)}_{\alpha\delta\zeta\eta}g_{\zeta\eta\mu\nu}(\epsilon+\epsilon')K^{(s)}_{\mu\nu\beta\rho}\right.\right.$$
$$\left.+\tfrac{1}{2}I^{(s)}_{\mu\alpha\nu\rho}L_{\nu\eta\mu\zeta}(\epsilon-\epsilon')I^{(s)}_{\delta\zeta\beta\eta}]\right)g_{\beta i}\left(\epsilon-\frac{\Omega}{2}\right)$$
$$\left.-\tfrac{1}{2}I^{(s)}_{\mu\alpha\nu\rho}\int d\omega\, L_{\nu\beta i\zeta}\left(\epsilon-\frac{\Omega}{2}+\frac{\omega}{2},\omega\right)g_{\eta\mu}\left(\omega+\epsilon'-\frac{\Omega}{2}\right)I^{(s)}_{\zeta\delta\beta\eta}\right\}. \tag{3.26}$$

Here the upper index s indicates that these forces are assumed to act statically (no energy dependence). Since the structure is very similar to (3.13) one can understand that sometimes the phenomenological treatment, where one starts with a hamiltonian in which the nucleon–nucleon potential is replaced by an effective potential, can be successful. In the more rigorous approach it turns out that one has to deal with three different kind of forces, namely v, $I^{(s)}$ and $K^{(s)}$. These forces agree only in first order. If one takes higher-order effects into account $I^{(s)}$ and $K^{(s)}$ differ from v and from each other. Furthermore in general $I^{(s)}$ and $K^{(s)}$ are complex potentials. This is even the case if one does not restrict the basis of states. Such a mathematical restriction leads to an additional change of these forces. Such complex forces have so far not been taken into account. The standard method is to use real forces for $I^{(s)}$ and $K^{(s)}$. Bes and Broglia (1971) for instance treated particle–vibration coupling in a semi-phenomenological manner using separable potentials.

As a general conclusion from our approximations we can deduce that the structure of the effective particle–hole interaction has essentially the following form:

$$I_{\alpha\delta\beta\rho}(\epsilon\epsilon',\Omega) = I^{(0)}(\epsilon,\Omega)+\sum_{p}\frac{I^{(1)}_{\alpha\delta\beta\rho}(\epsilon,\Omega)}{\epsilon'-\omega_p+i\eta}+\sum_{p'}\frac{I^{(2)}_{\alpha\delta\beta\rho}(\epsilon,\Omega)}{\epsilon'-\omega_{p'}-i\eta}, \tag{3.27}$$

where ω_p and $\omega_{p'}$ are functions of ϵ and Ω. The residues $I^{(1)}$ and $I^{(2)}$ can be deduced either from perturbation theory, ladder approximation or the solution of the p–p and p–h RPA problem, depending on the chosen approximation. Weakly energy-dependent terms can be incorporated into I^0.

4. Treatment of the asymmetric Green function

According to equation (2.9) the asymmetric Green function has the following spectral representation:

$$g^{N0}_{\rho\delta}(\epsilon) = \sum_{s}\frac{A^s_{\rho\delta}}{\epsilon-\epsilon_s+i\eta}+\sum_{r}\frac{B^r_{\rho\delta}}{\epsilon-\epsilon_r-i\eta}, \tag{4.1}$$

with

$$\epsilon_s = E^{N+1}-\frac{E_N+E_0}{2}\simeq\epsilon_\delta-\frac{E_N-E_0}{2}, \tag{4.2}$$

$$\epsilon_r = -E^{N-1}+\frac{E_N+E_0}{2}\simeq\epsilon_\rho+\frac{E_N-E_0}{2}, \tag{4.3}$$

$$A^s_{\rho\delta} = \langle N|a_\rho|N+1\rangle\langle N+1|a^\dagger_\delta|0\rangle, \tag{4.4}$$

$$B^r_{\rho\delta} = \langle N|a^\dagger_\delta|N-1\rangle\langle N-1|a_\rho|0\rangle. \tag{4.5}$$

The first-order approximation for ϵ_s and ϵ_r follows from commutator $[H, a_\rho]_-$ and $[H, a_\delta^\dagger]_-$, respectively. Utilizing equation (3.27) we obtain by complex integration:

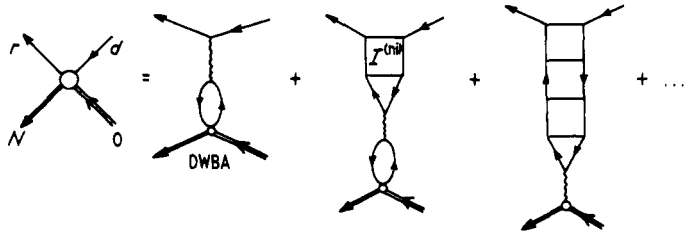
$$\sum_{\rho\delta}\int d\epsilon'\, I_{\alpha\delta\beta\rho}(\epsilon\epsilon', \Omega) g^{N0}_{\rho\delta}(\epsilon') = 2\pi i \sum_{\rho\delta}\Bigg(I^{(0)}_{\alpha\delta\beta\rho}(\epsilon, \Omega)\langle N|a_\rho^\dagger a_\delta|0\rangle + \sum_p \sum_r I^{(1)}_{\alpha\delta\beta\rho}(\epsilon, \Omega) B^r_{\rho\delta}(\epsilon_r - \omega_p + i\eta)^{-1} - \sum_{p'}\sum_s I^{(2)}_{\alpha\delta\beta\rho}(\epsilon, \Omega) A^s_{\rho\delta}(\epsilon_s - \omega_{p'} - i\eta)^{-1}\Bigg). \tag{4.6}$$

$I^{(1)}$, $I^{(2)}$, ω_p and $\omega_{p'}$ are known, if we assume knowledge of the forces and the result of nuclear structure calculations. From such calculations we can determine the explicit structure of $g^{(2)}(\omega)$ and $L(\omega)$. If we restrict ourselves to perturbation theory, we have to insert shell-model values for $g^{(2)}(\omega)$ and $L(\omega)$. The ϵ_s and ϵ_r can be taken either from experiment or from the solution for the two-point function, respectively. In the latter case one has to find, as discussed in § 2, a reasonable approximation for the energy-dependent part of the effective one-particle potential (MacKellar *et al* 1971, Dover and Giai 1971, Bruneau and Mau 1970, Werner 1967, Brenig 1967, Weigel 1968, Zawischa and Werner 1969, Vinh-Mau 1970, Winter 1972). We assume knowledge of $\epsilon_s(\epsilon_r)$ and the residues of the two-point function ($\langle N+1|a_\rho^\dagger|0\rangle, \ldots$ etc). For a complete solution we have to obtain the quantities $\langle N|a_\rho|N+1\rangle$ or $\langle N|a_\delta^\dagger|N-1\rangle$, respectively. From equation (2.16) we would be able to get an equation for these quantities (or a subset of them by restricting the number of states) utilizing the method of comparison of poles. The resulting sets of equations are complicated and it seems to us that a satisfactory solution is not obtainable. Therefore it seems necessary to look for an approximate solution of expression (4.6).

The first choice could be an iteration procedure starting with $I^{(ni)}$ in equation (2.28). This expansion seems reasonable, if the perturbation expansion of the whole process converges rapidly. Then we would obtain the following series:

$$g^{N0}_{\rho\delta}(\epsilon) = g^{N0}_{\rho\delta,\mathrm{DWBA}}(\epsilon) - 2\pi \sum_{\beta\alpha} L^0_{\rho\beta\delta\alpha}(\epsilon, \Omega) \sum_{\mu\nu}\int d\epsilon'\, I^{(ni)}_{\alpha\mu\beta\nu}(\epsilon\epsilon', \Omega) g^{N0}_{\nu\mu,\mathrm{DWBA}}(\epsilon') + (2\pi)^2 L^0_{\ldots}(\epsilon, \Omega)\Bigg(\int d\epsilon'\, I^{(ni)}_{\ldots}(\epsilon, \epsilon'; \Omega) L^0_{\ldots}(\epsilon', \Omega) \times \int d\epsilon''\, I^{(ni)}(\epsilon', \epsilon''; \Omega) g^{N0}_{\ldots,\mathrm{DWBA}}(\epsilon'')\Bigg) + \ldots. \tag{4.8}$$

In Feynman graphs this series shows the structure:



In the second choice we utilize our knowledge about the final state of the target nucleus. For instance we could assume a particle–hole structure of this state known

from a RPA calculation. Therefore we make the ansatz

$$|N\rangle = C_N^\dagger|0\rangle = \sum_{\text{ph}} (x_{\text{ph}}^N a_{\text{p}}^\dagger a_{\text{h}} - y_{\text{ph}}^N a_{\text{h}}^\dagger a_{\text{p}})|0\rangle \tag{4.9}$$

for the final state. The amplitudes x and y can be taken from a nuclear structure calculation (see, eg, Rowe 1970, Weigel *et al* 1971, Dover *et al* 1972). The essential problem consists now according to (4.4) and (4.5) in the determination of

$$\begin{aligned}\langle N|a_\rho|N+1\rangle &= \langle 0|C_N a_\rho|N+1\rangle \\ &= \sum_{\text{ph}} [(x_{\text{ph}}^N)^*\langle 0|a_{\text{h}}^\dagger a_{\text{p}} a_\rho|N+1\rangle - (y_{\text{ph}}^N)^*\langle 0|a_{\text{p}}^\dagger a_{\text{h}} a_\rho|N+1\rangle],\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}\langle N|a_\delta^\dagger|N-1\rangle &= \langle 0|C_N a_\delta^\dagger|N-1\rangle \\ &= \sum_{\text{ph}} [(x_{\text{ph}}^N)^*\langle 0|a_{\text{h}}^\dagger a_{\text{p}} a_\delta^\dagger|N-1\rangle - (y_{\text{ph}}^N)^*\langle 0|a_{\text{p}}^\dagger a_{\text{h}} a_\delta|N-1\rangle].\end{aligned} \tag{4.11}$$

One possibility for obtaining the matrix elements $\langle 0|a^\dagger aa|N+1\rangle$ and $\langle 0|a^\dagger aa^\dagger|N-1\rangle$ is to solve the so-called 2p–1h RPA problem, for which the theory is known (Winter 1972, Schuck *et al* 1973).

Another way to relate the problem to nuclear structure calculations consists in linking these matrix elements to the generalized density matrix of the odd-neighbour nuclei. This can be achieved by utilizing the commutator relations for fermions. We get for instance:

$$\langle 0|a_{\text{h}}^\dagger a_{\text{p}} a_\rho|N+1\rangle = -\delta_{\text{ph}}\langle 0|a_\rho|N+1\rangle + \sum_{|\overline{N+1}\rangle} \langle 0|a_\rho|\overline{N+1}\rangle\langle\overline{N+1}|a_{\text{h}}^\dagger a_{\text{p}}|N+1\rangle. \tag{4.12}$$

The generalized density matrix $\langle\overline{N\pm 1}|a_\alpha^\dagger a_\beta|N\pm 1\rangle$ can be calculated in several approximations discussed in Migdal (1965), Speth (1970), Weigel (1972). Essentially one has to know the structure of the autocorrelation function $L(\omega)$ of the density fluctuations. Experimentally the density matrix is related to moments and transitions. If we apply Migdal's quasiparticle approach $A_{\rho\delta}^s$ (or equivalently (4.12)) reduces in an obvious notation ($z_\alpha^{1/2} = \langle N+1, E_{\hat\alpha}, \hat\alpha|a_\alpha^\dagger|0\rangle, \ldots$; $\hat\alpha$ has to be a subset of the quantum number set α):

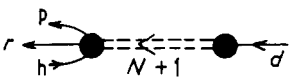
$$\begin{aligned}&\langle 0|a_{\text{p}}^\dagger a_{\text{h}} a_\rho|N+1, E_{\hat\mu}, \hat\mu\rangle\langle N+1, E_{\hat\mu}, \hat\mu|a_\mu^\dagger|0\rangle \\ &\simeq -\delta_{\rho\text{h}}(1-n_{\text{p}})(1-n_\mu) z_\mu^{1/2} z_{\text{p}}^{1/2}\delta_{\hat\mu,\hat\rho} + (1-n_\rho)(1-n_\mu) z_\rho^{1/2} z_\mu^{1/2}\langle N+1, E_{\hat\rho}, \hat\rho|a_{\text{h}}^\dagger a_{\text{p}}|N+1, E_{\hat\mu}, \hat\mu\rangle.\end{aligned} \tag{4.13}$$

In this method, the generalized density matrix can be directly related to the renormalized RPA problem with instantaneous forces. One obtains (Weigel 1972):

$$\begin{aligned}&\langle N+1, E_{\hat\rho}, \hat\rho|a_{\text{h}}^\dagger a_{\text{p}}|N+1, E_{\hat\mu}, \hat\mu\rangle \\ &= \delta_{\rho\mu}\langle 0|a_{\text{h}}^\dagger a_{\text{p}}|0\rangle + \delta_{\rho\text{h}}\delta_{\mu\text{p}} z_\rho^{1/2} z_\mu{}^{1/2} + 2\pi\sum_{\tau\lambda}\Bigg[-\tilde I_{\rho\lambda\mu\tau} \sum_{|N\rangle\neq|0\rangle}\Bigg(\frac{(\tilde y_{\text{ph}}^N)^*}{E_N - E_0 - E_{\hat\rho} - E_{\hat\mu} - \text{i}\eta} \\ &\qquad \times [(1-n_\lambda) n_\tau \tilde y_{\lambda\tau}^N + n_\lambda(1-n_\tau)\tilde x_{\tau\lambda}^N] \\ &\qquad + \frac{\tilde x_{\text{ph}}^N}{E_N - E_0 + E_{\hat\rho} + E_{\hat\mu} - \text{i}\eta}[(1-n_\lambda) n_\tau (\tilde x_{\lambda\tau}^N)^* + n_\lambda(1-n_\tau)(\tilde y_{\tau\lambda}^N)^*]\Bigg)\Bigg].\end{aligned} \tag{4.14}$$

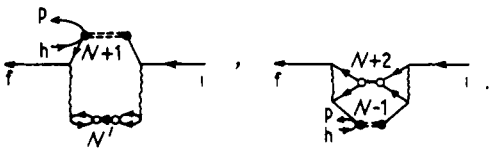
The quantities with a tilde denote the renormalized expressions (Weigel 1969, Weigel *et al* 1971). If we were to apply this method we could express $A^{s}_{\rho\delta}$ and $B^{r}_{\rho\delta}$ directly by known theoretical results.

It may be instructive to discuss the general result (4.6) in a schematic manner for some examples. If we use for instance the ansatz (4.9) we have to consider only graphs, which always have the vacuum state as the background. Then g^{N0}_{rd} has essentially the following form:



(4.15)

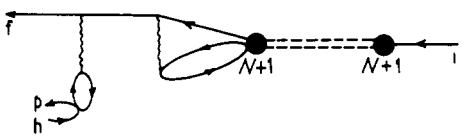
where we have drawn the first term of (4.1). Typical contributions to g^{N0}_{fi} from (4.6) show for instance the following structures:



(4.16)

Here, we presented two graphs emerging from (3.13). The first one is a contribution from the third term, the second one is caused by the second term in (3.13). If we use (3.26) we have to replace v by I or K, respectively. The interpretation of the diagrams is simple, the first process contains in the intermediate stage an excited state N' of the target nucleus plus a state of the $N+1$ system. If the energy of the outgoing particle is in the vicinity of $E_{N'}+E_{N+1}$ ($\epsilon_f \simeq \epsilon_\rho + E_{N'} - E_0$) we expect a relevant contribution to the scattering process. In the second graph we have the intermediate propagation of a state of the $N+2$ system plus a state of the $N-1$ system. Again we expect a larger contribution if energetically the intermediate stage is close to the in- or out-going situation. Whether such a possibility exists depends on the structure of the system considered. One can obtain under some approximations the explicit existence of a single Breit–Wigner resonance, if one restricts oneself to a single term in I. Additionally selection rules may play an important role.

One might expect additional terms, where a $N+1$ state alone propagates in a part of the intermediate structure, an example would be the next graph:



(4.17)

The right-hand part of this graph is not contained in $I(\epsilon, \epsilon', \Omega)$ but in the one-particle propagator. This can be seen easily by inversion of the equation of motion for the two-point function. In our presentation this term would be represented by τ^{D} (see (4.18)) via the relation:

$$\tau^{D}_{mm} g^{s}_{nb} = i v_{mxyr} g^{c}_{yrxb}, \tag{4.18}$$

if g^{s} is equal to the Hartree–Fock propagator. The total graph (4.17) is obtained if we use the energy-independent approximation for I (see (2.24)). Of course, one could use in principle insertions of the kind (4.16) on the left part (4.17), so obtaining higher-order processes.

5. Summary

By expressing the wavefunction of inelastic nucleon–nucleus scattering as a projected version of the generalized linear response function information about the intermediate structure of the process is contained in an integral over the irreducible vertex I times the asymmetric Green function† (see equations (2.17), (2.24)). An instantaneous effective particle–hole force leads directly to the well known DWBA approximation. Deviations from this approximation are therefore caused by the energy dependence of the irreducible vertex. Formally one can obtain a T matrix for the non-direct process given by (2.29). The structure of the irreducible vertex has been investigated in four approximations: (i) second-order (renormalized) perturbation theory; (ii) ladder approximation for the effective single-particle potential; (iii) linear approximation (in the effective scattering matrix) for the irreducible vertex; (iv) neglect of effective three-body interactions in the exact expression for the irreducible vertex.

For the purpose of obtaining explicit expressions for I one has to perform nuclear structure calculations of a different kind. One has to obtain the p–p or p–h structures of the target system by using the RPA approximations in different versions (first-order, standard or renormalized, respectively). The general outcome for I can be expressed as a sum (integral) of pole terms (see (3.27)). The energy integral was evaluated by utilizing the spectral representation of the asymmetric Green function. Three approximations have been considered: (i) the iteration procedure starting with the DWBA solution; (ii) use of the solution of the p–p–h RPA problem assuming knowledge of the final target state; (iii) by insertion of a complete set of $N \pm 1$ states, the problem can be reduced to the generalized density matrix of the odd-neighbour nuclei, for which several theories and calculations are available.

The integration was performed by Chauchy's theorem. The result shows the intermediate structure of the process discussed for some terms in more detail in § 4. Information mainly from nuclear structure calculations is needed in the expressions for the scattering matrix, so obtaining a connection between nucleon–nucleus scattering and the nuclear structure problem. The freedom of choice for the forces in the theory is restricted, since the same quantities ought to be used in different calculations.

Acknowledgments

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Appendix

Relevant for our problem is the system of equations for the two- and four-point functions. Application of the equation of motion for the field operators and Schwinger's principle lead to the following relations (Baym and Kadanoff 1962, Novozhilov and Tulub 1961, Naminiki 1960, Weigel 1969, Brenig and Wagner 1963) (L denotes the generalized

† It is possible to link the Green function approach to the coupled-channel treatment (see Winter 1974).

response function; q is the source term; $L^0_{msnr} \equiv \mathrm{i}g_{mr}g_{sn}$, and time dependence is included in v).

$$\mathrm{i}\frac{\delta g_{mn}}{\delta q_{rs}} = -\mathrm{i}(g_{msnr} - g_{mn}g_{sr}) \equiv L^0_{msnr} + L^0_{mdbc}\frac{\delta u_{cb}}{\delta q_{rs}}\bigg|_{q=0}$$

$$= L^0_{msnr} - L^0_{mdbc}I_{cqbp}L_{psqr}|_{q=0}, \tag{A.1}$$

$$u_{kl} = -2\mathrm{i}v_{kmln}g_{nm} + \mathrm{i}v_{krst}g_{sn}\frac{\delta u_{nl}}{\delta q_{rt}}\bigg|_{q=0}, \tag{A.2}$$

which can be cast in the following final form:

$$\left(\mathrm{i}\frac{\partial}{\partial t_k}\delta_{kn} - \frac{1}{2m}p^2_{kn} - u_{kn}\right)g_{nl} = \delta_{kl}, \tag{A.3}$$

$$u_{kj} = -2\mathrm{i}v_{kajb}g_{ba} + v_{kman}g_{ar}\Gamma_{rcjd}g_{dm}g_{nc}. \tag{A.4}$$

Γ denotes the effective scattering matrix in the medium obeying the following equation:

$$\Gamma_{a_1a_2a_1'a_2'} = \left(2v_{a_1b_2a_1'b_2'} + \mathrm{i}v_{a_1b_2c_1'c_2'}g_{c_1'c_1}g_{c_2'c_2}\Gamma_{c_1c_2a_1'b_2'} + 2\mathrm{i}v_{a_1c_2b_2'c_1'}g_{c_1'c_1}g_{c_2'c_2}\Gamma_{b_2c_1a_1'c_2'}\right.$$

$$\left. + \mathrm{i}v_{a_1c_2c_1'c_2'}g_{c_1'c_1}g_{c_2'n}\frac{\delta\Gamma_{c_1na_1'm}}{\delta g_{b_2'b_2}}g_{mc_2}\right) \times (\delta_{a_2b_2}\delta_{a_2'b_2'} - \mathrm{i}g_{b_2'd_2}g_{d_2'b_2}\Gamma_{d_2a_2d_2'a_2'})$$

$$= I_{a_1b_2a_2b_2'} \times (\ldots). \tag{A.5}$$

The connection between the reducible vertex Γ (effective scattering matrix) and the four-point function is given by:

$$g_{kmln} - g_{kl}g_{mn} + g_{kn}g_{ml} = \mathrm{i}g_{kr}g_{mu}\Gamma_{rusv}g_{vn}g_{sl} = g^c_{kmln} = \mathrm{i}g_{kr}g_{mu}K_{rusv}g_{vsnl}. \tag{A.6}$$

In the last expression the kernel is defined as the sum of irreducible graphs in the p–p channel (Nozieres 1964).

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